



# Bond Total Return Swaps

## Theory, Pricing & Practice



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## PART ONE: Theory

### Bond Total Return Swaps

- Introduction to Bond TRS
- Why Trade a TRS?
- Contract Specifications
- Constant Units vs Constant Notional

## PART TWO: Pricing & Practice

### Case Studies

- Pricing Formulae
- Bond TRS Case Study

Quant Research Papers

<https://ssrn.com/author=1728976>

Book: Low Latency Interest Rate Markets

<https://github.com/nburgessx/SwapsBook>



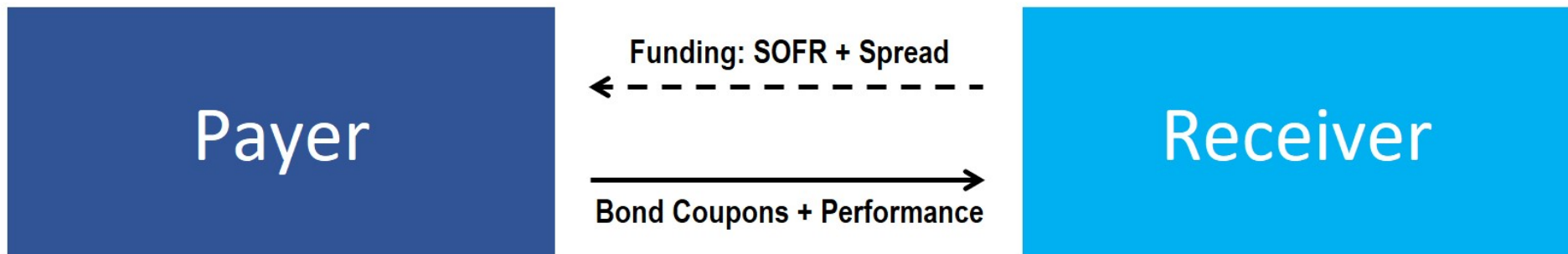
## PART ONE - THEORY

### Bond Total Return Swaps

# Bond TRS – Introduction to Bond TRS

## What is a Bond TRS?

- The funded purchase of a Bond, where we receive the **coupons** and **performance** cash flows
- The 'payer' passes all coupons and changes in Bond price are passed to the 'receiver'



## Cash Flows

- The payer holds the underlying bond, but passes on all **bond income** to receiver
- Bond income comes in the form of **bond coupons & capital appreciation** (bond performance)
- This means if bond prices rise (fall) the profit (loss) is passed to receiver
- In the event of a **default** the contract is unwound and the receiver pays the loss given default (LGD)

# Bond TRS – Why Trade a TRS?

## Pros

- **Capital Efficiency** - Much cheaper and less capital intensive than purchasing the bond directly
- **Leverage** – Capital efficiency helps increase leverage with minimal cash outlay
- **Market Access** – Direct market access not required



## Cons

- **Counterparty and Default Risk** – Exposed to both counterparty and bond default risk
- **Interest Rate Risk** – Interest rate changes impact both funding costs and bond returns
- **Regulatory Risk** – Off-balance sheet and may attract increased regulatory scrutiny and associated costs.



# Bond TRS – Trade Features

1. OTC, Customizable and Highly Bespoke
2. Fixed Cash Amount or Fixed Number of Bonds  
i.e. **Constant Notional** or **Constant Units**
3. Receive Bond **Coupons & Performance**
4. Pay Fixed or Float Funding
5. **Interest Rate & Credit Risk** (to Bond & Arranger)
6. Receiver Exposed to **Loss Given Default (LGD)**
7. Performance Projections Linked to Repo Rates
8. Pay Performance Regularly or At Maturity Only
9. **Regular Performance Payments Reduce PFE<sup>1</sup> Exposure** and XVA costs

The screenshot displays a financial software interface with a dark theme. The top navigation bar includes tabs for 91) Actions, 92) Products, 93) Views, 94) Info, and 95) Settings. Below this, there are sub-tabs for Solver (Premium), Load, Save, Trade, and CCP. The main content area is divided into several sections:

- Deal**: Includes fields for Basket Total Return, Counterparty, SWAP CNTRPARTY, and Ticker.
- Swap**: Contains details for Leg 1: Asset (Receive), Leg 2: Float (Pay), Notional (10MM), Currency (USD), Effective date (05/04/2020), Maturity (23Y 05/04/2043), Index (SOFR), Spread (0.000 bp), Leverage (1.00000), Latest Index (5.31000), Reset Freq (Daily), Pay Freq (Annual), and Day Count (ACT/360).
- Valuation Settings**: Includes Curve Date (05/13/2024), Valuation (05/13/2024), Calc Method (Accrual), and a checkbox for OIS DC Stripping.
- Market**: A section with a refresh icon.
- Leg 1: NPV**: Shows values for NPV (118,217.64), Accrued (118,217.64), and Premium (1.18).
- Leg 2: NPV**: Shows values for NPV (-13,281.98), Accrued (-13,281.98), and Premium (0.00).
- Valuation Results**: A table showing Principal (0.00), Accrued (104,935.66), NPV (104,935.66), Premium (0.00000), BP Value (104.93566), Capital Gain (-109,661.48), and Dividend Pmt (227,879.12).

<sup>1</sup> Potential Future Exposure

# Bond TRS – Contract Specifications Example

## Notional Specification

- 50,000 Bonds Purchased at 104.54.33%
- Cash Equivalent USD 5,227,165 (= 50,000 x 100 x 104.5433% )
- Bond Leg Specified as Number of Bonds (i.e. Units)

## Coupon Payments

- Bond Leg Receives Coupons & Performance
- Funding Leg Pays SOFR + 100 bps
- Performance Paid at Maturity Only
- Credit Risky i.e. Cash Flows Scaled by Survival Probability
- Pays Loss Given Default (LGD) = 1 – Recovery Rate

Leg	Bond Leg	Funding Leg
PayOrReceive	Receive	Pay
Notional	50,000	5,227,165
NotionalType	Units	Cash
NotionalExchange	No Exchanges	No Exchanges
Currency	USD	USD
UnderlyingBond	UST 6.0% 15-02-2026	
CashFlowType	Coupons + Performance	
FloatIndex		SOFR O/N Index
FixedRateOrFloatSpread		100 bps
PayCoupons	On Coupon Dates	
PayPerformance	At Maturity	
Leverage	1.00	1.00
IsCreditRisky	TRUE	TRUE
CreditCurve	US Treasury	US Treasury
IncludeLossGivenDefault	TRUE	
RecoveryRate	40%	
TradedBondPrice	104.5433%	
DiscountCurve	SOFR O/N Index	SOFR O/N Index
CreditCurve	US Treasury	

# Bond TRS – Constant Units

## Constant Units

- **Fixed Number of Bonds (Units)**
- **Variable Cash Notional over Time**
- **Every Period Cash Notional Exchanges**
- **Bond Fixings Required**

Constant Units

Time	Notional	Units	Bond Fixing	Performance	Coupon	Total Payment
Past	10,000,000	80,736	123.86	108,994	155,417	264,411
Past	10,108,994	80,736	125.21	-662,038	155,417	-506,620
Past	9,446,956	80,736	117.01	235,750	155,417	391,167
Past	9,682,706	80,736	119.93	-1,984,499	155,417	-1,829,081
Past	7,698,208	80,736	95.35	-919,587	155,417	-764,169
Past	6,778,621	80,736	83.96	668,497	155,417	823,914
Past	7,447,118	80,736	92.24	-796,867	155,417	-641,450
Past	6,650,250	80,736	82.37	169,546	155,417	324,964
<b>Current</b>	<b>6,819,797</b>	<b>80,736</b>	<b>84.47</b>	<b>-74,277</b>	<b>155,417</b>	<b>81,140</b>
Future	6,745,519	80,736	83.55	0	155,417	0
Future	6,745,519	80,736	83.55	0	155,417	0
Future	6,745,519	80,736	83.55	0	155,417	0
Future	6,745,519	80,736	83.55	0	155,417	0
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Future	6,745,519	80,736	83.55	0	155,417	0

$$\text{Units} = \left( \frac{\text{Cash Notional}}{\text{Face Value} \times \text{Bond Fixing}} \right)$$

$$\text{Units} = \left( \frac{10,000,000}{100 \times 123.86\%} \right) = 80,736$$

$$\text{Coupon} = \text{Units} \times \text{Face Value} \times \text{Rate} \times \text{Year Fraction}$$

$$\begin{aligned} \text{Coupon} &= 80,736 \times 100 \times 3.85\% \times 0.5 \\ &= 155,416.80 \end{aligned}$$



# Bond TRS – Constant Notional

## Constant Notional

- Fixed Cash Notional
- Variable Bond Units over Time
- No Notional Exchanges Required

Constant Notional

Time	Notional	Units	Bond Fixing	Performance	Coupon	Total Payment
Past	10,000,000	80,736	123.86	108,994	155,417	264,411
Past	10,000,000	79,866	125.21	-654,900	153,742	-501,158
Past	10,000,000	85,463	117.01	249,551	164,516	414,067
Past	10,000,000	83,382	119.93	-2,049,529	160,510	-1,889,019
Past	10,000,000	104,877	95.35	-1,194,546	201,888	-992,659
Past	10,000,000	119,104	83.96	986,184	229,276	1,215,460
Past	10,000,000	108,413	92.24	-1,070,035	208,695	-861,340
Past	10,000,000	121,403	82.37	254,947	233,702	488,649
Current	10,000,000	118,385	84.47	-108,914	227,892	118,977
Future	10,000,000	119,689	83.55	0	230,401	0
Future	10,000,000	119,689	83.55	0	230,401	0
Future	10,000,000	119,689	83.55	0	230,401	0
Future	10,000,000	119,689	83.55	0	230,401	0
Future	10,000,000	119,689	83.55	0	230,401	0
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Future	10,000,000	119,689	83.55	0	230,401	0

$$\text{Units} = \left( \frac{\text{Cash Notional}}{\text{Face Value} \times \text{Bond Fixing}} \right)$$

$$\text{Units} = \left( \frac{10,000,000}{100 \times 123.86\%} \right) = 80,736$$

$$\text{Coupon} = \text{Units} \times \text{Face Value} \times \text{Rate} \times \text{Year Fraction}$$

$$\begin{aligned} \text{Coupon} &= 80,736 \times 100 \times 3.85\% \times 0.5 \\ &= 155,416.80 \end{aligned}$$

## PART TWO – PRICING & PRACTICE

### Pricing & Case Study

## Bond TRS – Pricing Formula

Bond TRS has three main components,

- Underlying Bond Coupons
- Bond Performance
- Funding Component

The **Present Value** of a TRS is the Sum of **Discounted** Cash Flows from these components.

However, in the event of bond default the TRS is unwound and the receiver of the bond coupons must pay the **Loss Given Default (LGD)** to the bond holder i.e. the payer. The LGD must be included in the PV calculation.

## Bond TRS – Pricing

91) Actions ▾		92) Products ▾		93) Views ▾		94) Info ▾		95) Settings ▾			
Solver (Premium) ▾		Load		Save		Trade ▾		CCP ▾			
3) Main		4) Details		6) Cashflow		7) Resets					
Deal		Basket Total Return		Counterparty		SWAP CNTRPARTY ▾		+ Ticker / SWAP		20) Properties	
Swap		Receive ▾		Leg 2: Float		Pay ▾		Arrears		Valuation Settings	
Leg 1: Asset		Constant Notional ▾		Notional		10MM				Curve Date 05/13/2024	
Type		10MM		Currency		USD				Valuation 05/13/2024	
Notional		80,734.866698		Effective		-- 05/04/2020				Calc Method Accrual	
Unit		USD		Maturity		23Y 05/04/2043				<input checked="" type="checkbox"/> OIS DC Stripping	
Currency		05/04/2020		Index		1D SOFRRATE					
Effective		23Y 05/04/2043		Spread		0.000 bp					
Maturity		USD EJ659253@BMRK Cor		Leverage		1.00000					
Asset		84.474611		Latest Index		5.31000					
Previous Fixing		83.548250		Reset Freq		Daily					
Latest Value		SemiAnnual		Pay Freq		Annual					
Reset Freq		SemiAnnual		Day Count		ACT/360					
Pay Freq											
Market		↺									
Leg 1: NPV		118,217.64		Leg 2: NPV		-13,281.98					
Accrued		118,217.64		Accrued		-13,281.98					
Premium		1.18		Premium		0.00					
Valuation Results											
Principal		0.00		Premium		0.00000		Capital Gain		-109,661.48	
Accrued		104,935.66		BP Value		104.93566		Dividend Pmt		227,879.12	
NPV		104,935.66									
22) Calculators											

$$\text{Bond TRS PV} = \phi ( \text{PV}(\text{Bond Coupon}) + \text{PV}(\text{Performance}) - \text{PV}(\text{Funding Cash Flows}) )$$

where  $\phi = \{ +1 \text{ when receiving and } -1 \text{ when paying bond coupons} \}$

## Bond TRS – Pricing the Bond Coupons

### Coupon PV

- For simplicity let's define the quantity or **Bond Notional ( $N_B$ )** as Units x Face Value
- This gives **Coupon** = Bond Notional x Rate x Year Fraction
- Leading to **PV(Coupon)** = Bond Notional x Rate x Year Fraction x P(Survival) x Disc Factor
- Let's assume we have n Bond coupons

$$PV(\text{Bond Coupons}) = \sum_{i=1}^n N_B \cdot r \cdot \tau_i \cdot \underbrace{Q(t_0, t_i)}_{P(\text{Survive})} \cdot \underbrace{P(t_0, t_i)}_{\text{Disc Factor}}$$



# Bond TRS – Pricing the Bond Performance

## Bond Performance PV

- **Bond Performance has the largest impact on the TRS PV**
- The performance captures the Change in Bond Price using Bond Fixings (or resets)
- Change in Bond Price = Price at End of Coupon Period – Bond Price at Start of Coupon Period
- Leading to  $PV(\text{Performance}) = \text{Bond Notional} \times \text{Change in Price} \times P(\text{Survival}) \times \text{Disc Factor}$

$$PV(\text{Performance}) = \sum_{i=1}^n N_B \cdot (B(t_{i-1}) - B(t_i)) \cdot \underbrace{Q(t_0, t_i)}_{P(\text{Survive})} \cdot \underbrace{P(t_0, t_i)}_{\text{Disc Factor}} \quad \left. \vphantom{\sum_{i=1}^n} \right\} \text{When Paying Performance on Coupon Dates}$$

$$PV(\text{Performance}) = N_B \cdot (B(t_0) - B(T)) \cdot \underbrace{Q(t_0, T)}_{P(\text{Survive})} \cdot \underbrace{P(t_0, T)}_{\text{Disc Factor}} \quad \left. \vphantom{N_B} \right\} \text{When Paying Performance at Maturity, T}$$

# Bond TRS – Estimating or Projecting Future Bond Prices

## Projecting Future Bond Prices

- **Future Bond Prices can be implied from Repo Rates**
- The repo rate acts like a **Bond Growth Factor** as if we can place a bond on deposit and earn bond interest
- Repo rates work on **Dirty Bond Prices**

## Calculation Using Repo Rates

- The simple projected bond price is calculated as below,  
$$B_T = B_t(1 + r \tau)$$
- We should however deduct repo interest from coupons (c) paid during the projection period,  
$$B_T = B_t(1 + r \tau_1 - r c \tau_2)$$
- where  $B_t$  is the Dirty Bond Price at time t, r the repo rate, c the coupon amount paid,  $t_1$  denotes the time from t to T and  $t_2$  the time from the coupon payment to time T

# Bond TRS – Pricing the Funding Leg

## Funding Leg PV

- The client borrow funds to purchase the underlying bond
- In exchange for receiving bond coupons and performance we pay funding costs
- The funding leg has a **Cash Notional ( $N_C$ )**, not to be confused with the bond notional ( $N_B$ )
- Let's assume we are funding our position in USD using SOFR as the floating index plus a spread, denoted ( $F+s$ )
- Then the **Funding Cost** or **Float Coupon** = Cash Notional x (SOFR + Spread) x Year Fraction
- Leading to **PV(Funding Leg)** = Cash Notional x (SOFR + Spread) x Year Fraction x P(Survival) x Disc Factor
- Let's assume we have m floating coupons

$$PV(\text{Float Coupons}) = \sum_{j=1}^m N_C \cdot (F_j + s) \cdot \tau_j \cdot \underbrace{Q(t_0, t_j)}_{P(\text{Survive})} \cdot \underbrace{P(t_0, t_j)}_{\text{Disc Factor}}$$

## Bond TRS – TRS Pricing I

Putting Everything Together ...

$$\text{Bond TRS PV} = \phi ( \text{PV}(\text{Bond Coupon}) + \text{PV}(\text{Performance}) - \text{PV}(\text{Funding Cash Flows}) )$$

where  $\phi = \{ +1 \text{ when receiving and } -1 \text{ when paying bond coupons} \}$

$$\begin{aligned} & PV(\text{TRS}) \\ &= \phi \left( \underbrace{\sum_{i=1}^n N_B r \tau_i Q(t_0, t_i) P(t_0, t_i)}_{\text{Bond Coupons}} + \underbrace{\sum_{i=1}^n N_B (B(t_{i-1}) - B(t_i)) Q(t_0, t_i) P(t_0, t_i)}_{\text{Bond Performance}} - \underbrace{\sum_{j=1}^m N_C (F_j + s) \tau_j Q(t_0, t_j) P(t_0, t_j)}_{\text{Funding Cost}} \right) \end{aligned}$$

## Bond TRS – TRS Pricing II

### Definition: Risky Discount Factor

- A cash flow that can default is said to be **Credit Risky**
- To compute the PV of a credit risky cash flow we must discount the cash flow **AND** include the **Probability of Survival** i.e. the probability of no default so we can receive the cash flow.
- The **Risky Discount Factor** does just this and is defined as  $\tilde{P}(t_0, t_i) = Q(t_0, t_i) P(t_0, t_i)$

Using Risky Discount Factor Notation gives,

$$PV(\text{TRS}) = \varphi \left( \underbrace{\sum_{i=1}^n N_B r \tau_i \tilde{P}(t_0, t_i)}_{\text{Bond Coupons}} + \underbrace{\sum_{i=1}^n N_B (B(t_{i-1}) - B(t_i)) \tilde{P}(t_0, t_i)}_{\text{Bond Performance}} - \underbrace{\sum_{i=1}^n N_C (F_i + s) \tau_i \tilde{P}(t_0, t_i)}_{\text{Funding Cost}} \right)$$



## Bond TRS – TRS Pricing III

### What about the Loss Given Default?

- If the bond defaults the TRS is unwound and the **Loss Given Default** is paid by the bond coupon receiver
- We must incorporate the LGD defined as  $N_B(1-RR)$ , where RR is the recovery rate of the bond
- In developed markets the recovery rate is usually assumed to be 40% for senior subordinated bonds
- The probability of default during a coupon period is given by  $P(\text{Default}) = Q(t_0, t_{i-1}) - Q(t_0, t_i)$ , which is the probability of surviving to the start of the coupon period minus that of surviving to the end of the coupon period.

$$PV(\text{TRS}) = \varphi \left( \underbrace{\sum_{i=1}^n N_B r \tau_i \tilde{P}(t_0, t_i)}_{\text{Bond Coupons}} + \underbrace{\sum_{i=1}^n N_B (B(t_{i-1}) - B(t_i)) \tilde{P}(t_0, t_i)}_{\text{Bond Performance}} - \underbrace{\sum_{i=1}^n N_C (F_i + s) \tau_i \tilde{P}(t_0, t_i)}_{\text{Funding Cost}} - \underbrace{\sum_{i=1}^n N_B (1 - RR) (Q(t_0, t_{i-1}) - Q(t_0, t_i)) P(t_0, t_i)}_{\text{Loss Given Default}} \right)$$

## Bond TRS – The Breakeven Funding Spread (Par Spread)

### What is the Breakeven Funding Spread or Par Spread?

- This is the funding spread that gives a **TRS price of zero** or Par i.e. both trade legs have equal value
- To calculate the par spread we rearrange the equation for PV(TRS) for the funding cost “s” parameter
- We often rename the parameter “p” to indicate it is the par spread

The par spread is computed as,

$$p = \left( \frac{PV(TRS \text{ with } s = 0)}{N_C \tau_i \tilde{P}(t_0, t_i)} \right)$$

The term in the denominator is called the Risky Annuity,

$$Risky \text{ Annuity}(Float \text{ Leg}) = N_C \tau_i \tilde{P}(t_0, t_i)$$

Giving,

$$s = \left( \frac{PV(TRS \text{ with } s = 0)}{Risky \text{ Annuity } Float \text{ Leg}} \right)$$

## Appendix – Approximate Bond TRS Price I

### Bond TRS Approximate Pricing Methodology

We can compute an approximate Bond TRS PV as follows,

#### Steps:

1. **Bond Coupons** – Compute the PV of Bond Coupons within the TRS start and end dates
2. **Bond Performance** – Compute the Performance using Bond Prices as follows,
  - i. **Current Bond Price** - Price the Bond
  - ii. **Forward Bond Price** - Price the Bond again but with all coupons up to the TRS Maturity Date Removed
3. **Funding Leg** – Compute as Normal
4. **Loss Given Default** – Trivial to include, however for an approximate TRS PV we could perhaps ignore this for investment grade bonds i.e. assume  $\text{Prob}(\text{Default}) \approx 0$ .

# Appendix – Approximate Bond TRS Price II

What does the approximation look like when we net everything together?

1. Full Bond Cash Flows to Bond Maturity
2. Funding Leg has a Negative Notional Exchange Equal to the Current Bond Price Paid at TRS Maturity
3. Funding Leg to Maturity of TRS

## Illustration – Bond Leg Coupons and Performance

### Bond

Bond coupons plus final notional exchange

Bond Price 108.11  
Fwd Price 106.73

Time	Coupon	DF	PV
1	5	0.9615	4.81
2	5	0.9246	4.62
3	5	0.8890	4.44
4	5	0.8548	4.27
5	5	0.8219	4.11
6	5	0.7903	3.95
7	5	0.7599	3.80
8	5	0.7307	3.65
9	5	0.7026	3.51
10	105	0.6756	70.93

### TRS Coupons

Bond coupons are paid to the client  
Only include coupons during TRS lifetime of TRS

Total PV 9.43

Time	Coupon	DF	PV
1	5	0.9615	4.81
2	5	0.9246	4.62
3			
4			
5			
6			
7			
8			
9			
10			

### TRS Performance

Future bond price less initial price paid at maturity  
i.e. Future bond cashflows minus initial bond PV

Total PV -1.27

Time	Coupon	DF	PV
1		0.9615	0.00
2	-108.11	0.9246	-99.95
3	5	0.8890	4.44
4	5	0.8548	4.27
5	5	0.8219	4.11
6	5	0.7903	3.95
7	5	0.7599	3.80
8	5	0.7307	3.65
9	5	0.7026	3.51
10	105	0.6756	70.93

### TRS

Bond coupons plus performance

Total PV 8.16

Time	Coupon	DF	PV
1	5	0.9615	4.81
2	-103.11	0.9246	-95.33
3	5	0.8890	4.44
4	5	0.8548	4.27
5	5	0.8219	4.11
6	5	0.7903	3.95
7	5	0.7599	3.80
8	5	0.7307	3.65
9	5	0.7026	3.51
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Have questions or want further info?

## Contact

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